1.B.1 Since $y \succ z$ implies $y \succsim z$, the transitivity implies that $x \succsim z$. Suppose that $z \succsim x$. Since $y \succsim z$, the transitivity then implies that $y \succsim x$. But this contradicts $x \succ y$. Thus we cannot have $z \succsim x$. Hence $x \succ z$.

1.B.2 By the completeness, $x \succsim x$ for every $x \in X$. Hence there is no $x \in X$ such that $x \succ x$. Suppose that $x \succ y$ and $y \succ z$, then $x \succ y \succsim z$. By (iii) of Proposition 1.B.1, which was proved in Exercise 1.B.1, we have $x \succ z$. Hence $\succ$ is transitive. Property (i) is now proved.

As for (ii), since $x \succsim x$ for every $x \in X$, $x \sim x$ for every $x \in X$ as well. Thus $\sim$ is reflexive. Suppose that $x \sim y$ and $y \sim z$. Then $x \succsim y$, $y \succsim z$, $y \succsim x$, and $z \succsim y$. By the transitivity, this implies that $x \succsim z$ and $z \succsim x$. Thus $x \sim z$. Hence $\sim$ is transitive. Suppose x that $\sim y$. Then $x \succsim y$ and $y \succsim x$. Thus $y \succsim x$ and $x \succsim y$. Hence $y \sim x$. Thus $\sim$ is symmetric. Property (ii) is now proved.

3.B.2 Suppose that $x \gg y$. Define $\varepsilon = \text{Min}\{x_1 - y_1, \ldots, x_L - y_L\} > 0$, then, for every $z \in X$, if $\|y - z\| < \varepsilon$, then $x \gg z$. By the local nonsatiation, there exists $z^* \in X$ such that $\|y - z^*\| < \varepsilon$ and $z^* \succ y$. By $x \gg z^*$ and the weak monotonicity, $x \succsim z^*$. By Proposition 1.B.1(iii) (which is implied by the transitivity), $x \succ y$. Thus $\succsim$ is monotone.

3.C.2 Take a sequence of pairs $\{(x^n, y^n)\}_{n=1}^{\infty}$ such that $x^n \succsim y^n$ for all n, $x^n \to x$, and $y^n \to y$. Then $u(x^n) \geq u(y^n)$ for all n, and the continuity of $u(\cdot)$ implies that $u(x) \geq u(y)$. Hence $x \succsim y$. Thus $\succsim$ is continuous.

3.C.5 (a) Suppose first that $u(\cdot)$ is homogeneous of degree one and let $\alpha \geq 0$, $x \in \mathbb{R}^L_+$, $y \in \mathbb{R}^L_+$, and $x \sim y$. Then $u(x) = u(y)$ and hence $\alpha u(x) = \alpha u(y)$. By the homogeneity, $u(\alpha x) = u(\alpha y)$. Thus $\alpha x \sim \alpha y$.

Suppose conversely that $\succsim$ is homothetic. We shall prove that the utility function constructed in the proof of Proposition 3.C.1 is homogeneous of degree one. Let $x \in \mathbb{R}^L_+$ and $\alpha > 0$, then $u(x)e \sim x$ and $u(\alpha x)e \sim \alpha x$. Since $\succsim$ is homothetic, $\alpha u(x)e \sim \alpha x$. By the transitivity of $\sim$ (Proposition 1.B.1(ii)), $u(\alpha x)e \sim \alpha u(x)e$. Thus $u(\alpha x) = \alpha u(x)$.